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Anomalies on orbifolds with gauge symmetry breaking

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Abstract: We embed two 4D chiral multiplets of opposite representations in the 5D $N = 2$ SU$(N + K)$ gauge theory compactified on an orbifold $S^1/(Z_2 \times Z_2')$. There are two types of orbifold boundary conditions in the extra dimension to obtain the 4D $N = 1$ SU(N) $\times$ SU(K) $\times$ U(1) gauge theory from the bulk: in type-I, one has the bulk gauge group at $y = 0$ and the unbroken gauge group at $y = \pi R/2$ while in type-II, one has the unbroken gauge group at both fixed points. In both types of orbifold boundary conditions, we consider the zero mode(s) as coming from a bulk $(K + N)$-plet and brane fields at the fixed point(s) with the unbroken gauge group. We check the consistency of this embedding of fields by the localized anomalies and the localized FI terms. We show that the localized anomalies in type-I are cancelled exactly by the introduction of a bulk Chern-Simons term. On the other hand, in some class of type-II, the Chern-Simons term is not enough to cancel all localized anomalies even if they are globally vanishing. We also find that for the consistent embedding of brane fields, there appear only the localized log FI terms at the fixed point(s) with a U(1) factor.

Keywords: Field Theories in Higher Dimensions, Anomalies in Field and String Theories.

Contents

1. Introduction

Recently the orbifold unification models in the existence of extra dimensions have drawn much attention due to their simplicity in performing the gauge symmetry breaking and the doublet-triplet splitting at the same time. The unwanted zero modes appearing in the unification models are projected out by boundary conditions in the extra dimension, i.e, they get masses of order of the compactification scale. For instance, the Minimal Supersymmetric Standard Model(MSSM) fields were obtained in the 5D SUSY SU(5) model where the extra dimension is compactified on a simple orbifold $S^1/(Z_2 \times Z_2')$ [1, 2, 3]. The idea was also taken in the model with the 5D SU(3) electroweak unification with the TeV-sized extra dimension [4, 5], the possibility of which was first considered in the context of the string orbifolds [6].

In the orbifold with gauge symmetry breaking, in general, in addition to the fixed point where the bulk gauge symmetry is operative, there exists a fixed point where only the unbroken gauge group is respected [2]: for instance, $G_{SM} = \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ in the case with the 5D SUSY SU(5) model on $S^1/(Z_2 \times Z_2')$. Therefore, we can put a multiplet(so called a brane field) at that fixed point allowed by the representation of the unbroken gauge group. In the more realisic model constructions, there has been a lot of various possibilities of having incomplete multiplets located at the orbifold fixed points (or branes) with an unbroken gauge group [2]–[5]. For instance, in the 5D SU(5) GUT on $S^1/(Z_2 \times Z_2')$, it has been shown that the $s-\mu$ puzzle can be understood from the introduction of a split multiplet for **10** of the second generation [3] while the top-bottom mass hierarchy can be also explained with one Higgs in the bulk and the other Higgs at the brane [3, 5]. Moreover, introducing incomplete multiplets for the quark setor is indispensible in the 5D SU(3) electroweak unification on $S^1/(Z_2 \times Z_2')$ [4, 5].

However, with incomplete multiplets, there could appear the localized gauge anomalies on the independent orbifold boundaries [7, 8, 9, 10, 5, 11]. It has been shown [12] that the abelian anomalies coming from a bulk field in 5D are equally distributed at the fixed points by the half of its 4D anomaly while the bulk Chern-Simons term [13] plays a role in conveying localized anomaly at one fixed point to the other fixed point. So, the 4D anomaly cancellation for zero modes is sufficient for consistency. The anomaly analysis can be generalized to the case in the non-abelian gauge anomalies on the orbifold with gauge symmetry breaking. It has been shown in 5D SU(5) and SU(3) orbifold unification models [5] that for fermion zero modes of the 4D anomaly-free combination, the integrated anomalies are absent but the localized anomalies can be exactly cancelled by the introduction of a bulk Chern-Simons counter term.

In the 4D supersymmetric theory, it is known that the Fayet-Iliopoulos term(FI) also can be radiatively generated only for anomalous U(1) gauge theories. This FI term could break supersymmetry and/or anomalous U(1). However, the situation is somewhat different in orbifold models with a U(1) factor. In 5D S^1/Z_2 or $S^1/(Z_2\times Z_2')$ orbifold with a U(1) factor [14, 9, 11, 17], it has been recently shown that there is a possibility that the integrated FI term vanishes while there exist non-zero localized FI terms at the fixed points. These localized FI terms, however, do not affect either 4D supersymmetry or gauge symmetry since they can be absorbed by the non-vanishing vacuum expectation value of a real scalar field belonging to the bulk vector multiplet [15, 16, 9, 11, 17]. The net effect of the localized FI terms is the dynamical localization of the bulk zero mode and to make the bulk massive modes decoupled. Even in the case where quadratically divergent FI terms are cancelled locally by introducing at each fixed point a brane field with the half charge compared to that of a bulk field, the *logarithmically divergent* FI terms are equally distributed at both fixed points, which also gives rise to the localization of the zero mode at both fixed points [11].

In this paper, we consider the gauge symmetry breaking due to the boundary conditions in the 5D $N=2$ SUSY $G=\mathrm{SU}(N+K)$ gauge theory on $S^1/(Z_2\times Z_2')$. In type-I, we have the full gauge symmetry G at $y=0$ and the unbroken gauge symmetry $H=\mathrm{SU}(N)\times\mathrm{SU}(K)\times\mathrm{U}(1)$ at $y=\pi R/2$. On the other hand, in type-II, we have the unbroken gauge group at both fixed points. When we consider the zero modes coming from a bulk $(N+K)$-plet in both types of orbifold boundary conditions, we show that the bulk fermion gives rise to the localized gauge anomalies: $+\frac{1}{2}$ of G^3 gauge anomalies at $y=0$ and $+\frac{1}{2}$ of H^3 gauge anomalies at $y=\pi R/2$ in type-I while H^3 gauge anomalies are equally split at both fixed points in type-II. Using this result, we find that addition of one $\bar{K}$-plet located at $y=\pi R/2$ in type-I leads to $-\frac{1}{2}$ of H^3 gauge anomalies at $y=\pi R/2$ while there remains $+\frac{1}{2}$ of G^3 gauge anomalies at $y=0$. However, we also show that all the localized gauge anomalies are cancelled exactly by introducing a Chern-Simons(CS) 5-form with a jumping coefficient in the 5D action [13, 12, 8]. On the other hand, in type-II with brane fields at the same fixed point, we show that a bulk CS term is not sufficient for cancelling the localized gauge anomalies.

Secondly, we consider the localized FI terms in our model. In type-I, the localized FI terms could appear only at $y=\pi R/2$ with a U(1) factor. Since we have the $N=1$ supersymmetry without integrated gauge/gravitational anomalies at the zero mode level, the integrated FI term should vanish for consistency. Therefore, it can be argued that the FI term should be absent locally in our model [5]. On the other hand, in type-II, there could appear the FI terms at both fixed points, which seems to be not necessarily zero with the condition for no integrated FI term. However, since there is no gravitational counterpart of CS term in GUT orbifolds, there should not be localized gravitational anomalies, which allows only a specific assigning of brane fields. In this paper, we show that there exists only a non-vanishing log divergent FI term at $y = \pi R/2$ in type-I and log divergent FI terms at both fixed points in a consistent class of type-II, which still makes the integrated FI term to be zero. The log divergent FI term can be absorbed by a singular vacuum expectation value(VEV) of the U(1) gauge component of the real adjoint scalar field in the bulk without changing the mass spectrum [11].

Our paper is organized as follows. In the next section, we give an introduction to the gauge symmetry breaking on orbifolds by adopting the 5D SUSY SU$(N+K)$ gauge theory on $S^1/(Z_2 \times Z_2')$. Then, in the section 3, for this GUT orbifold, we derive the detailed expression for the localized non-abelian anomalies coming from a bulk fermion in the fundamental representation of SU$(N+K)$. The section 4 is devoted to the localization problem of a bulk fermion and the cancellation of the localized gauge anomalies. In the next two sections, we work out with the localized Fayet-Iliopoulos terms in our model. Then, we conclude the paper in the last section.

2. Orbifold breaking of gauge symmetry

Let us consider the five-dimensional SUSY $G = \mathrm{SU}(N+K)$ gauge theory compactified on an $S^1/(Z_2 \times Z_2')$ orbifold. The fifth dimensional coordinate y is compactified to a circle $2\pi R \equiv 0$. Furthermore, the point $y = -a$ is identified to $y = a$ (Z_2 symmetry) and the point $y = (\pi R/2)+a$ is identified to $y = (\pi R/2)-a$ (Z_2' symmetry). Then, the fundamental region of the extra dimension becomes the interval $[0, \pi R/2]$ between two fixed points $y = 0$ and $y = \pi R/2$.

For the two Z_2 symmetries, one can define their actions P and P' within the configuration space of any bulk field:

$$\phi(x,y) \to \phi(x,-y) = P\phi(x,y)\,, \tag{2.1}$$

$$\phi(x,y') \to \phi(x,-y') = P'\phi(x,y') \tag{2.2}$$

where $y' \equiv y + \pi R/2$. The (P, P') actions can involve all the symmetries of the bulk theory, for instance, the gauge symmetry and the R-symmetry in the supersymmetric case. In general, then, any bulk field ϕ can take one of four different Fourier expansions depending on their pair of two Z_2 parities, (i,j) as

$$\phi_{++} = \sum_{n=0}^{\infty} \sqrt{\frac{1}{2^{\delta_{n,0}}\pi R}}\phi^{(2n)}_{++}(x^\mu)\cos\frac{2ny}{R} \tag{2.3}$$

$$\phi_{+-} = \sum_{n=0}^{\infty} \sqrt{\frac{1}{\pi R}}\phi^{(2n+1)}_{+-}(x^\mu)\cos\frac{(2n+1)y}{R} \tag{2.4}$$

$$\phi_{-+} = \sum_{n=0}^{\infty} \sqrt{\frac{1}{\pi R}} \phi_{-+}^{(2n+1)}(x^\mu) \sin \frac{(2n+1)y}{R} \tag{2.5}$$

$$\phi_{--} = \sum_{n=0}^{\infty} \sqrt{\frac{1}{\pi R}} \phi_{--}^{(2n+2)}(x^\mu) \sin \frac{(2n+2)y}{R} \tag{2.6}$$

where x^μ is the 4D space-time coordinate.

The minimal supersymmetry in 5D corresponds to N=2 supersymmetry(or 8 supercharges) in the 4D N=1 language. Thus, a 5D chiral multiplet corresponds to an N=2 hypermultiplet consisting of two N=1 chiral multiplets with opposite charges. Two 4D Weyl spinors make up one 5D spinor. On the other hand, a 5D vector multiplet corresponds to an N=2 vector multiplet composed of one N=1 vector multiplet($V = (A_\mu, \lambda_1, D) \equiv V^q T^q$) and one N=1 chiral multiplet ($\Sigma = ((\Phi + iA_5)/\sqrt{2}, \lambda_2, F_\Sigma) \equiv \Sigma^q T^q$), which transforms in the adjoint representation of the bulk gauge group.[1] Upon compactification, we consider the case where one Z_2 breaks N=2 supersymmetry to N=1 while the other Z_2 breaks the bulk $G = \mathrm{SU}(N+K)$ gauge group to its subgroup $H = \mathrm{SU}(N) \times \mathrm{SU}(K) \times \mathrm{U}(1)$.

For instance, a bulk hypermultiplet in the fundamental of $\mathrm{SU}(N+K)$, which is composed of two chiral multiplets with opposite charges, $H = (h, \psi, F_H) \equiv (H_1, H_2)^T$ and $H^c = (h^c, \psi^c, F_{H^c}) \equiv (H_1^c, H_2^c)$, transforms under Z_2 and Z_2' identifications as

$$H(x,-y) = \eta P H(x,y)\,, \qquad H^c(x,-y) = -\eta H^c(x,y) P^{-1} \tag{2.7}$$

$$H(x,-y') = \eta' P' H(x,y')\,, \qquad H^c(x,-y') = -\eta' H^c(x,y') P'^{-1} \tag{2.8}$$

where both η and η' can take $+1$ or -1, and $P^2 = P'^2 = I_{N+K}$ where I_{N+K} is the $(N+K) \times (N+K)$ identity matrix. On the other hand, the bulk gauge multiplet is transformed under the two Z_2 transformations respectively as

$$V(x,-y) = P V(x,y) P^{-1}\,, \tag{2.9}$$

$$\Sigma(x,-y) = -P \Sigma(x,y) P^{-1}\,, \tag{2.10}$$

$$V(x,-y') = P' V(x,y') P'^{-1}\,, \tag{2.11}$$

$$\Sigma(x,-y') = -P' \Sigma(x,y') P'^{-1}\,. \tag{2.12}$$

Now let us consider two types of the parity assignment (P, P') as

$$\text{type I}: \quad P = I_{N+K}\,, \quad P' = \mathrm{diag}(I_K, -I_N)\,, \tag{2.13}$$

$$\text{type II}: \quad P = P' = \mathrm{diag}(I_K, -I_N)\,. \tag{2.14}$$

For both types of parity matrices, the $G = \mathrm{SU}(N+K)$ gauge symmetry is broken down to $H = \mathrm{SU}(N) \times \mathrm{SU}(K) \times \mathrm{U}(1)$ because P'(P also for type-II) does not commute with all the gauge generators of $\mathrm{SU}(N+K)$: $P' T^a P'^{-1} = T^a$ and $P' T^{\hat{a}} P'^{-1} = -T^{\hat{a}}$ where $q = (a, \hat{a})$ denote unbroken and broken generators, respectively. However, the fixed point gauge groups are different: for type-I, the G bulk gauge symmetry at $y = 0$ and the H

[1] We note $D = X_3 - \partial_5 \Sigma$ and $F_\Sigma = (X_1 + iX_2)/\sqrt{2}$ in terms of the $\mathrm{SU}(2)_R$ triplet $\vec{X}$ in $N = 2$ supersymmetry.

unbroken gauge symmetry at $y = \pi R/2$; for type-II, the H unbroken gauge symmetry at both $y = 0$ and $y = \pi R/2$. Therefore, in either case, it is possible to put some incomplete multiplets transforming only under the unbroken gauge group at the fixed point(s).

For type-I with $\eta = \eta' = 1$, the bulk hypermultiplet in the fundamental of $\mathrm{SU}(N+K)$ is split as follows

$$H_1^{(2n)}: \quad \left[(++); \left(1, K, \frac{1}{K}\right)\right], \quad \text{mass} = \frac{2n}{R} \tag{2.15}$$

$$H_2^{(2n+1)}: \quad \left[(+-); \left(N, 1, -\frac{1}{N}\right)\right], \quad \text{mass} = \frac{(2n+1)}{R} \tag{2.16}$$

$$H_2^{c(2n+1)}: \quad \left[(-+); \left(\overline{N}, 1, \frac{1}{N}\right)\right], \quad \text{mass} = \frac{(2n+1)}{R} \tag{2.17}$$

$$H_1^{c(2n+2)}: \quad \left[(--); \left(1, \overline{K}, -\frac{1}{K}\right)\right], \quad \text{mass} = (2n+2)R \tag{2.18}$$

where the brackets [ ] contain the quantum numbers of $Z_2 \times Z_2' \times \mathrm{SU}(N) \times \mathrm{SU}(K) \times \mathrm{U}(1)$. Consequently, upon compactification, there appears a zero mode only from the K-plet among the bulk field components while other fields get massive. Therefore, the bulk vector multiplet of $\mathrm{SU}(N+K)$ is divided into the different KK modes

$$V^{a(n)}: \quad [(++); (N^2-1, 1) + (1, K^2-1) + (1,1)], \quad \text{mass} = \frac{2n}{R} \tag{2.19}$$

$$V^{\hat{a}(2n+1)}: [(+-); (N, K) + (\overline{N}, \overline{K})], \quad \text{mass} = \frac{(2n+1)}{R} \tag{2.20}$$

$$\Sigma^{\hat{a}(2n+1)}: [(-+); (N, K) + (\overline{N}, \overline{K})], \quad \text{mass} = \frac{(2n+1)}{R} \tag{2.21}$$

$$\Sigma^{a(2n+2)}: [(--); (N^2-1, 1) + (1, K^2-1) + (1,1)], \quad \text{mass} = \frac{(2n+2)}{R} \tag{2.22}$$

where the brackets [ ] contain the quantum numbers of $Z_2 \times Z_2' \times \mathrm{SU}(N) \times \mathrm{SU}(K)$. Therefore, the orbifolding retains only the $\mathrm{SU}(N) \times \mathrm{SU}(K) \times \mathrm{U}(1)$ gauge multiplets as massless modes $V^{a(0)}$ while the KK massive modes for unbroken and broken gauge bosons are paired up separately.

For type-II with $\eta = \eta' = 1$, likewise, the bulk hypermultiplet is split as follows

$$H_1^{(2n)}: \quad \left[(++); \left(1, K, \frac{1}{K}\right)\right], \quad \text{mass} = \frac{2n}{R} \tag{2.23}$$

$$H_2^{(2n+2)}: \quad \left[(--); \left(N, 1, -\frac{1}{N}\right)\right], \quad \text{mass} = \frac{(2n+2)}{R} \tag{2.24}$$

$$H_2^{c(2n)}: \quad \left[(++); \left(\overline{N}, 1, \frac{1}{N}\right)\right], \quad \text{mass} = \frac{2n}{R} \tag{2.25}$$

$$H_1^{c(2n+2)}: \quad \left[(--); \left(1, \overline{K}, -\frac{1}{K}\right)\right], \quad \text{mass} = \frac{(2n+2)}{R}. \tag{2.26}$$

Therefore, in this case, there appears a zero mode of $\bar{N}$-plet as well as a zero mode of K-plet. On the other hand, the bulk vector multiplet of $\mathrm{SU}(N+K)$ is divided into the different KK modes

$$V^{a(n)}: \quad [(++);(N^2-1,1)+(1,K^2-1)+(1,1)]\,, \quad \text{mass} = \frac{2n}{R} \tag{2.27}$$

$$V^{\hat{a}(2n+2)}: [(--);(N,K)+(\overline{N},\overline{K})]\,, \ \text{mass} = \frac{(2n+2)}{R} \tag{2.28}$$

$$\Sigma^{\hat{a}(2n)}: \quad [(++);(N,K)+(\overline{N},\overline{K})]\,, \quad \text{mass} = \frac{2n}{R} \tag{2.29}$$

$$\Sigma^{a(2n+2)}: [(--);(N^2-1,1)+(1,K^2-1)+(1,1)]\,, \quad \text{mass} = \frac{(2n+2)}{R}\,. \tag{2.30}$$

Thus, on top of the zero mode of the H gauge multiplet, there exists a zero mode of the chiral multiplet $\Sigma^{\hat{a}}$ containing the broken gauge component of A_5. This new zero mode gets a radiative mass of order of the compactification scale and it can be identified with a Higgs multiplet, for instance, in the bulk SU(3) gauge theory.

Since each component of a gauge parameter $\omega = \omega^q T^q$ has the same Z_2 parities as those of the corresponding gauge field, the bulk gauge transformation is restricted as follows

$$\delta A^a_M = \partial_M \omega^a + i f^{abc} A^b_M \omega^c + i f^{a\hat{b}\hat{c}} A^{\hat{b}}_M \omega^{\hat{c}}\,, \tag{2.31}$$

$$\delta A^{\hat{a}}_M = \partial_M \omega^{\hat{a}} + i f^{\hat{a}\hat{b}c} A^{\hat{b}}_M \omega^c + i f^{\hat{a}b\hat{c}} A^b_M \omega^{\hat{c}} \tag{2.32}$$

where $f^{ab\hat{c}}$ and $f^{\hat{a}\hat{b}\hat{c}}$ are put to zero for the parity invariance. Particularly, since $\omega^{\hat{a}}$ takes the same parities $(+,-)((-,-))$ as $A^{\hat{a}}_\mu$ in type-I(II), the gauge transformation at $y=\pi R/2$(at both $y=0$ and $y=\pi R$) becomes the one of the unbroken gauge group H from eq. (2.31).

3. Non-abelian anomalies on orbifolds

A 5D fermion is not chiral in the 4D language. However, after orbifold compactification of the extra dimension, a chiral fermion can be obtained as the zero mode of a bulk non-chiral fermion. Then, the chiral fermion gives rise to the 4D gauge anomaly after integrating out the extra dimension. For the case with the 5D U(1) gauge theory on S^1/Z_2 [12] or $S^1/(Z_2\times Z_2')$ [7], it was shown that the 4D gauge anomaly coming from a zero mode is equally distributed at the fixed points. In this section, we do the anomaly analysis in the case with the 5D SU$(N+K)$ gauge theory compactified on our gauge symmetry breaking orbifold, $S^1/(Z_2\times Z_2')$.

Let us consider a four-component bulk fermion in the fundamental representation of SU$(N+K)$. Then, the action is

$$S = \int d^4x \int_0^{2\pi R} dy\, \bar{\psi}(i\not{D} - \gamma_5 D_5 - m(y))\psi \tag{3.1}$$

where $\not{D} = \gamma^\mu D_\mu$ and $D_M = \partial_M + iA_M$. Here $m(y)$ is a mass term for the bulk fermion and $A_M = A^q_M T^q$ is a classical non-abelian gauge field.

With the assignments of Z_2 and Z_2' parities to a $(N+K)$-plet hypermultiplet in the previous section, the fermion field transforms as

$$\psi(y) = \gamma_5 P \psi(-y)\,, \qquad \psi(y') = \gamma_5 P' \psi(-y') \tag{3.2}$$

where P and P' are given by eqs. (2.13) and (2.14), acting in the group space. Invariance of the action under two Z_2's gives rise to the conditions for the mass function

$$m(y) = -m(-y)\,, \qquad m(y') = -m(-y')\,. \tag{3.3}$$

And the gauge fields also transform under Z_2 as

$$A_\mu(y) = PA_\mu(-y)P^{-1}\,, \qquad A_5(y) = -PA_5(-y)P^{-1}\,, \tag{3.4}$$

and we replace ($y \to y'$, $P \to P'$) for Z_2' action.

Then, with $\psi = \psi^1 + \psi^2$, where 1 and 2 denotes K-plet and N-plet components respectively, the fermion field is decomposed into four independent chiral components

$$\psi^1 = \psi^1_L + \psi^1_R\,, \qquad \psi^2 = \psi^2_L + \psi^2_R \tag{3.5}$$

where

$$\gamma_5\psi^1_{L(R)} = \pm\psi^1_{L(R)}\,, \qquad \gamma_5\psi^2_{L(R)} = \pm\psi^2_{L(R)}\,. \tag{3.6}$$

First let us consider the case with type-I parity assignments. Due to the parity assignments, $(\pm,\pm)$ for $\psi^1_{L(R)}$ and $(\pm,\mp)$ for $\psi^2_{L(R)}$, we can expand each Weyl fermion in terms of KK modes

$$\psi^1_{L(R)}(x,y) = \sum_n \psi^1_{L(R)n}(x)\xi_n^{(\pm\pm)}(y)\,, \tag{3.7}$$

$$\psi^2_{L(R)}(x,y) = \sum_n \psi^2_{L(R)n}(x)\xi_n^{(\pm\mp)}(y)\,, \tag{3.8}$$

with

$$(-\partial_5 + m(y))(\partial_5 + m(y))\xi_n^{(+\pm)}(y) = M_n^2\xi_n^{(+\pm)}(y)\,, \tag{3.9}$$

$$(\partial_5 + m(y))(-\partial_5 + m(y))\xi_n^{(-\mp)}(y) = M_n^2\xi_n^{(-\mp)}(y) \tag{3.10}$$

where M_n is the nth KK mass. Here we note that ξ's make an orthonormal basis for the function on $[0, 2\pi R)$:

$$\int_0^{2\pi R} dy\,\xi_m^{(\pm\pm)}(y)\xi_n^{(\pm\pm)}(y) = \int_0^{2\pi R} dy\,\xi_m^{(\pm\mp)}(y)\xi_n^{(\pm\mp)}(y) = \delta_{mn}\,, \tag{3.11}$$

$$\int_0^{2\pi R} dy\,\xi_m^{(++)}(y)\xi_n^{(\pm\mp)}(y) = \int_0^{2\pi R} dy\,\xi_m^{(--)}(y)\xi_n^{(\pm\mp)}(y) = 0\,. \tag{3.12}$$

Under the gauge $A_5 = 0$,[2] inserting the mode sum of the fermion into the 5D action, we obtain

$$S = \int d^4x \Bigg[\sum_n \overline{\psi^1_n}(i\partial\!\!\!/ - M_{2n})\psi^1_n + \sum_n \overline{\psi^2_n}(i\partial\!\!\!/ - M_{2n-1})\psi^2_n \\ - \sum_{m,n}\Big(V_{mn}(A^a) + V_{mn}(A^i) + V_{mn}(B) + V_{mn}(A^{\hat a}) \Big) \Bigg] \tag{3.13}$$

[2]The result will be not changed in the case without a gauge condition [9].

where $\psi_n^1 = \psi_{Ln}^1 + \psi_{Rn}^1$ for $n > 0$ ($\psi_0^1 = \psi_{L0}^1$), $\psi_n^2 = \psi_{Ln}^2 + \psi_{Rn}^2$, and V_{mn}'s denote gauge vertex couplings. The $G = \mathrm{SU}(N+K)$ gauge fields($A = A^q T^q$) can be decomposed into

$$(N+K)^2 - 1 \to (N^2-1, 1) + (1, K^2-1) + (1,1) + (N,K) + (\overline{N}, \overline{K}) \,, \qquad (3.14)$$

that is, $A^a T^a (a = 1, \cdots, N^2-1)$, $A^i T^i (i = 1, \cdots, K^2-1)$, $A^{(N+K)^2-1} T^{(N+K)^2-1} \equiv BT^B$ gauge fields for the $H = \mathrm{SU}(N) \times \mathrm{SU}(K) \times \mathrm{U}(1)$ group, and $A^{\hat{a}}(t^{\hat{a}})_{\alpha r} \equiv X^{\alpha r} (\alpha = 1, \cdots, N; r = 1, \cdots, K)$ gauge fields for the G/H coset space, respectively. Here, broken group generators are related to $t^{\hat{a}}$ as

$$T^{\hat{a}} \equiv \begin{pmatrix} 0 & t^{\hat{a}} \\ (t^{\hat{a}})^\dagger & 0 \end{pmatrix} . \qquad (3.15)$$

Then, V_{mn}'s are given by the following:

$$\begin{aligned}
V_{mn}(A^a) &= J^{\mu a}_{mn(+-)} \mathcal{A}^{a(+-)}_{mn\mu} + J^{\mu a}_{mn(-+)} \mathcal{A}^{a(-+)}_{mn\mu} \\
V_{mn}(A^i) &= J^{\mu i}_{mn(++)} \mathcal{A}^{i(++)}_{mn\mu} + J^{\mu i}_{mn(--)} \mathcal{A}^{a(--)}_{mn\mu} \\
V_{mn}(B) &= J^{\mu B}_{mn(++)} \mathcal{B}^{(++)}_{mn\mu} + J^{\mu B}_{mn(--)} \mathcal{B}^{(--)}_{mn\mu} + J^{\mu B}_{mn(+-)} \mathcal{B}^{(+-)}_{mn\mu} + J^{\mu B}_{mn(-+)} \mathcal{B}^{(-+)}_{mn\mu} \\
V_{mn}(A^{\hat{a}}) &= J^{\mu \hat{a}}_{mn(+)} \mathcal{A}^{\hat{a}(+)}_{mn\mu} + J^{\mu \hat{a}}_{mn(-)} \mathcal{A}^{\hat{a}(-)}_{mn\mu}
\end{aligned} \qquad (3.16)$$

where

$$\mathcal{A}^{a(\pm\mp)}_{mn\mu} = \int_0^{2\pi R} dy\, \xi_m^{(\pm\mp)}(y) \xi_n^{(\pm\mp)}(y) A^a_\mu(x,y) \,, \qquad (3.17)$$

$$\mathcal{A}^{i(\pm\pm)}_{mn\mu} = \int_0^{2\pi R} dy\, \xi_m^{(\pm\pm)}(y) \xi_n^{(\pm\pm)}(y) A^i_\mu(x,y) \,, \qquad (3.18)$$

$$\mathcal{B}^{(\pm\pm)}_{mn\mu} = \int_0^{2\pi R} dy\, \xi_m^{(\pm\pm)}(y) \xi_n^{(\pm\pm)}(y) B_\mu(x,y) \,, \qquad (3.19)$$

$$\mathcal{B}^{(\pm\mp)}_{mn\mu} = \int_0^{2\pi R} dy\, \xi_m^{(\pm\mp)}(y) \xi_n^{(\pm\mp)}(y) B_\mu(x,y) \,, \qquad (3.20)$$

$$\mathcal{A}^{\hat{a}(\pm)}_{mn\mu} = \int_0^{2\pi R} dy\, \xi_m^{(\pm\mp)}(y) \xi_n^{(\pm\pm)}(y) A^{\hat{a}}_\mu(x,y) \qquad (3.21)$$

and

$$J^{\mu a}_{mn(\pm\mp)} = \overline{\psi^2_m} \gamma^\mu P_\pm T^a \psi^2_n \,, \qquad J^{\mu i}_{mn(\pm\pm)} = \overline{\psi^1_m} \gamma^\mu P_\pm T^i \psi^1_n \,, \qquad (3.22)$$

$$J^{\mu B}_{mn(\pm\pm)} = \overline{\psi^1_m} \gamma^\mu P_\pm T^B_{K\times K} \psi^1_n \,, \qquad J^{\mu B}_{mn(\pm\mp)} = \overline{\psi^2_m} \gamma^\mu P_\pm T^B_{N\times N} \psi^2_n \,, \qquad (3.23)$$

$$J^{\mu \hat{a}}_{mn(\pm)} = \overline{\psi^2_m} \gamma^\mu P_\pm t^{\hat{a}} \psi^1_n + \overline{\psi^1_m} \gamma^\mu P_\pm (t^{\hat{a}})^\dagger \psi^2_n \qquad (3.24)$$

with $P_\pm = (1 \pm \gamma_5)/2$. Here a decomposition of T^B is understood such as $T^B = \mathrm{diag}(T^B_{N\times N}, T^B_{K\times K})$. We note that the chiral current for the $\mathrm{SU}(N+M)$ gauge symmetry is split into chiral currents coupled to the unbroken and broken gauge fields.

Applying the classical equations of motion and the standard results for the 4D chiral anomalies [12, 8, 9], we can derive the anomalies for the chiral currents classified above.

By making an inverse Fourier-transformation by the convolution of the bulk eigenmodes, the 5D gauge vector current $J^{Mq} = \overline{\psi}\gamma^M T^q \psi$ is given by

$$
\begin{aligned}
J^{\mu a}(x,y) &= \sum_{m,n} (\xi_m^{(+-)}\xi_n^{(+-)} J^{\mu a}_{mn(+-)} + \xi_m^{(-+)}\xi_n^{(-+)} J^{\mu a}_{mn(-+)})\,, &(3.25)\\
J^{\mu i}(x,y) &= \sum_{m,n} (\xi_m^{(++)}\xi_n^{(++)} J^{\mu i}_{mn(++)} + \xi_m^{(--)}\xi_n^{(--)} J^{\mu i}_{mn(--)})\,, &(3.26)\\
J^{\mu B}(x,y) &= \sum_{m,n} (\xi_m^{(++)}\xi_n^{(++)} J^{\mu B}_{mn(++)} + \xi_m^{(--)}\xi_n^{(--)} J^{\mu B}_{mn(--)} + \\
&\quad + \xi_m^{(+-)}\xi_n^{(+-)} J^{\mu B}_{mn(+-)} + \xi_m^{(-+)}\xi_n^{(-+)} J^{\mu B}_{mn(-+)})\,, &(3.27)\\
J^{\mu \hat{a}}(x,y) &= \sum_{m,n} (\xi_m^{(+-)}\xi_n^{(++)} J^{\mu \hat{a}}_{mn(+)} + \xi_m^{(-+)}\xi_n^{(--)} J^{\mu \hat{a}}_{mn(-)}) &(3.28)
\end{aligned}
$$

and we can construct J^{5q} similarly. Consequently, it turns out that the divergence of the 5D gauge vector current is given in terms of the 4D gauge anomalies as

$$
\begin{aligned}
(D_M J^M)^a(x,y) &= f_2(y)(\mathcal{Q}^a(A) + \mathcal{Q}^a(X))\,, &(3.29)\\
(D_M J^M)^i(x,y) &= f_1(y)(\mathcal{Q}^i(A) + \mathcal{Q}^i(X))\,, &(3.30)\\
(D_M J^M)^B(x,y) &= f_1(y)(\mathcal{Q}^B_+(A) + \mathcal{Q}^B_+(X)) + f_2(y)(\mathcal{Q}^B_-(A) + \mathcal{Q}^B_-(X))\,, &(3.31)\\
(D_M J^M)^{\hat{a}}(x,y) &= f_1(y)(\mathcal{Q}^{\hat{a}}_1(X) + \mathcal{Q}^{\hat{a}}_+(X)) + f_2(y)(\mathcal{Q}^{\hat{a}}_2(X) + \mathcal{Q}^{\hat{a}}_-(X)) &(3.32)
\end{aligned}
$$

where

$$
\begin{aligned}
f_1(y) &= \sum_n \left[(\xi_n^{(++)}(y))^2 - (\xi_n^{(--)}(y))^2\right] = \frac{1}{4}\sum_n \delta(y - \frac{n\pi R}{2})\,, &(3.33)\\
f_2(y) &= \sum_n \left[(\xi_n^{(+-)}(y))^2 - (\xi_n^{(-+)}(y))^2\right] = \frac{1}{4}\sum_n (-1)^n \delta(y - \frac{n\pi R}{2})\,. &(3.34)
\end{aligned}
$$

The localized gauge anomalies $\mathcal{Q}$'s are composed of two large parts: anomalies for unbroken group components and broken group components of the 5D vector current. The anomalies for unbroken group components involve not only unbroken gauge fields

$$
\begin{aligned}
\mathcal{Q}^a(A) &= \frac{1}{32\pi^2}(D^{abc}F^b_{\mu\nu}\tilde{F}^{c\mu\nu}(x,y) + D^{abB}F^b_{\mu\nu}\tilde{F}^{B\mu\nu}(x,y))\,, &(3.35)\\
\mathcal{Q}^i(A) &= \frac{1}{32\pi^2}D^{ijB}F^j_{\mu\nu}\tilde{F}^{B\mu\nu}(x,y) + \frac{1}{32\pi^2}D^{ijk}F^j_{\mu\nu}\tilde{F}^{k\mu\nu}(x,y)\,, &(3.36)\\
\mathcal{Q}^B_+(A) &= \frac{1}{32\pi^2}\mathrm{Tr}\,(T^B_{K\times K})^3 F^B_{\mu\nu}\tilde{F}^{B\mu\nu}(x,y) + \\
&\quad + \frac{1}{64\pi^2}\mathrm{Tr}\,(\{T^B_{K\times K}, T^i\}T^j)F^i_{\mu\nu}\tilde{F}^{j\mu\nu}(x,y)\,, &(3.37)\\
\mathcal{Q}^B_-(A) &= \frac{1}{32\pi^2}\mathrm{Tr}\,(T^B_{N\times N})^3 F^B_{\mu\nu}\tilde{F}^{B\mu\nu}(x,y) + \\
&\quad + \frac{1}{64\pi^2}\mathrm{Tr}\,(\{T^B_{N\times N}, T^a\}T^b)F^a_{\mu\nu}\tilde{F}^{b\mu\nu}(x,y)\,, &(3.38)\\
\mathcal{Q}^B_+(A) + \mathcal{Q}^B_-(A) &= \frac{1}{32\pi^2}(D^{BBB}F^B_{\mu\nu}\tilde{F}^{B\mu\nu}(x,y) + D^{Bij}F^i_{\mu\nu}\tilde{F}^{j\mu\nu}(x,y) + \\
&\quad + D^{Bab}F^a_{\mu\nu}\tilde{F}^{b\mu\nu}(x,y)) \equiv \mathcal{Q}^B(A)\,, &(3.39)
\end{aligned}
$$

but also broken gauge fields

$$\begin{aligned}
\mathcal{Q}^a(X) &= \frac{1}{64\pi^2}\mathrm{Tr}\,(T^a t^{\hat{b}}(t^{\hat{c}})^\dagger)F^{\hat{b}}_{\mu\nu}\tilde{F}^{\hat{c}\mu\nu}(x,y) = \frac{1}{32\pi^2}D^{a\hat{b}\hat{c}}F^{\hat{b}}_{\mu\nu}\tilde{F}^{\hat{c}\mu\nu}(x,y)\,, && (3.40)\\
\mathcal{Q}^i(X) &= \frac{1}{64\pi^2}\mathrm{Tr}\,(T^i (t^{\hat{b}})^\dagger t^{\hat{c}})F^{\hat{b}}_{\mu\nu}\tilde{F}^{\hat{c}\mu\nu}(x,y) = \frac{1}{32\pi^2}D^{i\hat{b}\hat{c}}F^{\hat{b}}_{\mu\nu}\tilde{F}^{\hat{c}\mu\nu}(x,y)\,, && (3.41)\\
\mathcal{Q}^B_+(X) &= \frac{1}{64\pi^2}\mathrm{Tr}\,(\{T^B,(t^{\hat{b}})^\dagger\}t^{\hat{c}})F^{\hat{b}}_{\mu\nu}\tilde{F}^{\hat{c}\mu\nu}(x,y)\,, && (3.42)\\
\mathcal{Q}^B_-(X) &= \frac{1}{64\pi^2}\mathrm{Tr}\,(\{T^B,t^{\hat{b}}\}(t^{\hat{c}})^\dagger)F^{\hat{b}}_{\mu\nu}\tilde{F}^{\hat{c}\mu\nu}(x,y)\,, && (3.43)\\
\mathcal{Q}^B_+(X)+\mathcal{Q}^B_-(X) &= \frac{1}{32\pi^2}D^{B\hat{b}\hat{c}}F^{\hat{b}}_{\mu\nu}\tilde{F}^{\hat{c}\mu\nu}(x,y) \equiv \mathcal{Q}^B(X)\,. && (3.44)
\end{aligned}$$

On the other hand, the anomalies for broken group components of the 5D vector current become

$$\begin{aligned}
\mathcal{Q}^{\hat{a}}_1(X) &= \frac{1}{64\pi^2}\mathrm{Tr}\,((\{t^{\hat{a}},(t^{\hat{b}})^\dagger\}+\{(t^{\hat{a}})^\dagger,t^{\hat{b}}\})T^a)F^{\hat{b}}_{\mu\nu}\tilde{F}^{a\mu\nu}(x,y)\\
&= \frac{1}{32\pi^2}D^{\hat{a}\hat{b}a}F^{\hat{b}}_{\mu\nu}\tilde{F}^{a\mu\nu}(x,y), && (3.45)\\
\mathcal{Q}^{\hat{a}}_2(X) &= \frac{1}{64\pi^2}\mathrm{Tr}\,((\{t^{\hat{a}},(t^{\hat{b}})^\dagger\}+\{(t^{\hat{a}})^\dagger,t^{\hat{b}}\})T^i)F^{\hat{b}}_{\mu\nu}\tilde{F}^{i\mu\nu}(x,y)\\
&= \frac{1}{32\pi^2}D^{\hat{a}\hat{b}i}F^{\hat{b}}_{\mu\nu}\tilde{F}^{i\mu\nu}(x,y)\,, && (3.46)\\
\mathcal{Q}^{\hat{a}}_+(X) &= \frac{1}{64\pi^2}\mathrm{Tr}\,(((t^{\hat{a}})^\dagger t^{\hat{b}}+(t^{\hat{b}})^\dagger t^{\hat{a}})T^B_{K\times K})F^{\hat{b}}_{\mu\nu}\tilde{F}^{B\mu\nu}(x,y)\\
&= \frac{1}{64\pi^2}\mathrm{Tr}\,(\{T^{\hat{a}},T^{\hat{b}}\}T^B_{K\times K})F^{\hat{b}}_{\mu\nu}\tilde{F}^{B\mu\nu}(x,y)\,, && (3.47)\\
\mathcal{Q}^{\hat{a}}_-(X) &= \frac{1}{64\pi^2}\mathrm{Tr}\,((t^{\hat{a}}(t^{\hat{b}})^\dagger+t^{\hat{b}}(t^{\hat{a}})^\dagger)T^B_{N\times N})F^{\hat{b}}_{\mu\nu}\tilde{F}^{B\mu\nu}(x,y)\\
&= \frac{1}{64\pi^2}\mathrm{Tr}\,(\{T^{\hat{a}},T^{\hat{b}}\}T^B_{N\times N})F^{\hat{b}}_{\mu\nu}\tilde{F}^{B\mu\nu}(x,y)\,, && (3.48)\\
\mathcal{Q}^{\hat{a}}_+(X)+\mathcal{Q}^{\hat{a}}_-(X) &= \frac{1}{32\pi^2}D^{\hat{a}\hat{b}B}F^{\hat{b}}_{\mu\nu}\tilde{F}^{B\mu\nu}(x,y) \equiv \mathcal{Q}^{\hat{a}}_3(X)\,. && (3.49)
\end{aligned}$$

In all the expressions for the anomalies above, we note that D^{abc} denotes the symmetrized trace of group generators

$$D^{abc} = \frac{1}{2}\mathrm{Tr}\,(\{T^a,T^b\}T^c) \qquad (3.50)$$

and other D symbols with different group idices are similarly understood.

As a result, we find that a bulk fermion gives rise to the localized gauge anomalies for all gauge components of the 5D vector current. Since the broken gauge fields vanish at $y = \pi R/2$ due to their boundary conditions, the localized gauge anomalies at $y = \pi R/2$ are only $\mathcal{Q}(A)$'s, i.e., the H^3 gauge anomalies. However, at the other fixed point $y = 0$, in addition to $\mathcal{Q}(A)$'s, there also appear the localized gauge anomalies $\mathcal{Q}(X)$'s, so we obtain the localized anomalies of G^3 at $y = 0$. Restricting to the region $[0, 2\pi R)$, we can rewrite the divergence of the 5D vector current as

$$(D_M J^M)^a(x,y) = \frac{1}{2}\left(\delta(y) - \delta\left(y - \frac{\pi R}{2}\right)\right)\mathcal{Q}^a(A) + \frac{1}{2}\delta(y)\mathcal{Q}^a(X)\,, \qquad (3.51)$$

$$(D_M J^M)^i(x,y) = \frac{1}{2}\left(\delta(y)+\delta\left(y-\frac{\pi R}{2}\right)\right)\mathcal{Q}^i(A)+\frac{1}{2}\delta(y)\mathcal{Q}^i(X)\,, \tag{3.52}$$

$$(D_M J^M)^B(x,y) = \frac{1}{2}\left(\delta(y)+\delta\left(y-\frac{\pi R}{2}\right)\right)\mathcal{Q}^B_+(A)+\frac{1}{2}\left(\delta(y)-\delta\left(y-\frac{\pi R}{2}\right)\right)\mathcal{Q}^B_-(A)$$
$$+\frac{1}{2}\delta(y)\mathcal{Q}^B(X)\,, \tag{3.53}$$

$$(D_M J^M)^{\hat{a}}(x,y) = \frac{1}{2}\delta(y)\mathcal{Q}^{\hat{a}}(X) \tag{3.54}$$

where $\mathcal{Q}^{\hat{a}}(X) \equiv \mathcal{Q}^{\hat{a}}_1(X)+\mathcal{Q}^{\hat{a}}_2(X)+\mathcal{Q}^{\hat{a}}_3(X)$.

For the case with type-II parity assignments, making the mode expansions of the bulk fermion as

$$\psi^1_{L(R)}(x,y) = \sum_n \psi^1_{L(R)n}(x)\xi^{(\pm\pm)}_n(y)\,, \tag{3.55}$$

$$\psi^2_{L(R)}(x,y) = \sum_n \psi^2_{L(R)n}(x)\xi^{(\mp\mp)}_n(y) \tag{3.56}$$

which make up a Dirac fermion at each KK level as $\psi^1_n=\psi^1_{Ln}+\psi^1_{Rn}$ and $\psi^2_n=\psi^2_{Ln}+\psi^2_{Rn}$ for $n>0(\psi^1_0=\psi^1_{L0}$ and $\psi^2_0=\psi^2_{R0})$, and following the similar procedure as before, a bulk fermion gives rise to the divergence of the 5D vector current as

$$(D_M J^M)^a(x,y) = -\frac{1}{2}\left(\delta(y)+\delta\left(y-\frac{\pi R}{2}\right)\right)\mathcal{Q}^a(A)\,, \tag{3.57}$$

$$(D_M J^M)^i(x,y) = \frac{1}{2}\left(\delta(y)+\delta\left(y-\frac{\pi R}{2}\right)\right)\mathcal{Q}^i(A)\,, \tag{3.58}$$

$$(D_M J^M)^B(x,y) = \frac{1}{2}\left(\delta(y)+\delta\left(y-\frac{\pi R}{2}\right)\right)(\mathcal{Q}^B_+(A)-\mathcal{Q}^B_-(A))\,, \tag{3.59}$$

$$(D_M J^M)^{\hat{a}}(x,y) = 0\,. \tag{3.60}$$

Therefore, we find that there is no localized anomalies related to X,Y gauge bosons and the H anomalies coming from the zero modes of K-plet and $\bar{N}$-plet are equally distributed on the orbifold fixed points.

4. Localization of a bulk field and anomaly problem

As shown in the section 2, we can freely put some brane fields consistently with the local gauge symmetries at the fixed points: a brane field at $y=0$ should be a representation of SU$(N+K)$ while a brane field at $y=\pi R/2$ should be a representation of SU$(N)\times$ SU$(K)\times$ U(1). Since we assume that a bulk fermion gives rise to a K-plet as the zero mode and we want to have the anomaly-free theory at least at the zero mode level, we can only put a brane field of $\bar{K}$-plet at $y=\pi R/2$. This introduction of an incomplete brane multiplet is sufficient for the 4D anomaly-free theory at low energies but it could be inconsistent due to the existence of the localized gauge anomalies on the boundaries of the extra dimension. In this section, we consider the localization of a bulk fermion with a kink mass and subsequently deal with the appearing anomaly problem by using the results in the previous section.

It was shown in the literature that the localization of a bulk fermion can be realized by introducing a kink mass in the lagrangian and even a brane fermion is possible in the limit of a kink mass being infinite [12]. In the 5D U(1) gauge theory on S^1/Z_2 with a single bulk fermion, as a result of introducing an infinite kink mass, the anomaly contribution from a bulk fermion on the boundaries of the extra dimension was interpreted as the sum of contributions from a brane fermion and a parity-violating Chern-Simon term in 5D [12]. In other words, as a kink mass becomes infinite, heavy KK modes are decoupled but their effects remain as a local counterterm such as the 5D Chern-Simon term. The similar observation has been made for the non-abelian anomalies on orbifolds [5].

In our case with gauge symmetry breaking on orbifolds, an infinite kink mass, depending on its sign, could give rise to the localization of the unwanted bulk modes as massless modes [9, 17, 18] in type-I. For instance, a positive(negative) infinite kink mass for the even modes ($(+,+)$ and $(-,-)$) gives rise to a localization of the massless mode for $(+,+)$ at $y=0(y=\pi R/2)$. On the other hand, a positive infinite kink mass for the odd modes ($(+,-)$ and $(-,+)$) could lead to new massless modes localized at $y=0$ and $y=\pi R/2$, respectively. Suppose that there are the universal(preserving the bulk gauge symmetry) kink masses for even and odd modes, i.e., $m(y)=M\epsilon(y)I_{(N+K)\times(N+K)}$ in eq. (3.1) where $\epsilon(y)$ is the sign function with periodicity πR. Then, in order to avoid unwanted massless modes in type-I, we only have to take the sign of M to be negative. For instance, when we introduce a bulk multiplet $\overline{(N+K)}$ with $M\to-\infty$, we obtain a massless $\bar{K}$-plet only from the $(+,+)$ mode, which is localized at $y=\pi R/2$, while other modes get decoupled from the theory. Thus, in this respect, a brane $\bar{K}$-plet is naturally realized from a bulk complete multiplet in the field theoretic limit. In this process of localization, we find that the consistency with the incomplete brane field can be guaranteed with introducing a 5D Chern-Simons term [5], which would be interpreted as the effects from the decoupled heavy modes [13, 12, 8]. On the other hand, in type-II, a bulk $\overline{(N+K)}$ with $M>0(M<0)$ also gives rise to the localization of two zero modes at different branes: the zero mode of $\bar{K}$-plet at $y=0(y=\pi R/2)$ and the zero mode of N-plet at $y=\pi R/2(y=0)$. However, in this case, it is not possible to localize the two brane fields at the same brane in a field theory. This is related to the inconsistency of a class of type-II in view of localized gauge and gravitational anomalies which are not cancelled by the CS contributions as will be shown later.

Let us consider the local anomaly cancellation in both types I and II of GUT orbifolds. To begin with, in type-I, we can introduce a brane $\bar{K}$-plet at $y=\pi R/2$ on top of the bulk $(N+K)$-plet. Then, it gives rise to 4D gauge anomalies such as $-\mathcal{Q}^i(A)$ and $-\mathcal{Q}^B_+(A)$ at that fixed point, which cancel the 4D global anomalies coming from the zero mode of the bulk K-plet. Therefore, the resultant divergence of the 5D vector current is changed to

$$(D_MJ^M)^a(x,y)=\frac{1}{2}\bigg(\delta(y)-\delta\Big(y-\frac{\pi R}{2}\Big)\bigg)\mathcal{Q}^a(A)+\frac{1}{2}\delta(y)\mathcal{Q}^a(X)\,, \tag{4.1}$$

$$(D_MJ^M)^i(x,y)=\frac{1}{2}\bigg(\delta(y)-\delta\Big(y-\frac{\pi R}{2}\Big)\bigg)\mathcal{Q}^i(A)+\frac{1}{2}\delta(y)\mathcal{Q}^i(X)\,, \tag{4.2}$$

$$(D_MJ^M)^B(x,y)=\frac{1}{2}\bigg(\delta(y)-\delta\Big(y-\frac{\pi R}{2}\Big)\bigg)\mathcal{Q}^B(A)+\frac{1}{2}\delta(y)\mathcal{Q}^B(X)\,, \tag{4.3}$$

$$(D_M J^M)^{\hat{a}}(x,y) = \frac{1}{2}\delta(y)\mathcal{Q}^{\hat{a}}(X)\,. \tag{4.4}$$

Here we observe that the total localized gauge anomalies only involving the unbroken gauge group($\mathcal{Q}(A)$'s) appear in the combination of $(\delta(y)-\delta(y-\pi R/2))$, so their integrated gauge anomalies vanish. On the other hand, the anomalies involving broken gauge fields($\mathcal{Q}(X)$'s) remain nonzero even after integration because $\mathcal{Q}(X)$'s are nonzero only at $y=0$. This asymmetric localization of $\mathcal{Q}(X)$'s reflects the difference between two fixed point groups in type-I.

On the other hand, in type-II, we can introduce two brane incomplete fields($\bar{K}$-plet and N-plet) for no global anomalies in different ways since both fixed points have only the unbroken gauge group operative. Two brane fields could be located at the same brane or different branes. First let us consider the case that two brane fields are located at different branes, i.e. $\bar{K}$-plet at $y=\pi R/2(y=0)$ and N-plet at $y=0(y=\pi R/2)$. Then, the nonvanishing localized anomalies become

$$(D_M J^M)^{a}(x,y) = \pm\frac{1}{2}\left(\delta(y)-\delta\left(y-\frac{\pi R}{2}\right)\right)\mathcal{Q}^{a}(A)\,, \tag{4.5}$$

$$(D_M J^M)^{i}(x,y) = \pm\frac{1}{2}\left(\delta(y)-\delta\left(y-\frac{\pi R}{2}\right)\right)\mathcal{Q}^{i}(A)\,, \tag{4.6}$$

$$(D_M J^M)^{B}(x,y) = \pm\frac{1}{2}\left(\delta(y)-\delta\left(y-\frac{\pi R}{2}\right)\right)\mathcal{Q}^{B}(A)\,. \tag{4.7}$$

Thus, the integrated anomalies vanish as expected but there exist localized gauge anomalies involving the unbroken gauge group. Secondly, in case that two brane fields are located at the same branes $y=0(y=\pi R)$, we get the total localized anomalies as

$$(D_M J^M)^{a}(x,y) = \pm\frac{1}{2}\left(\delta(y)-\delta\left(y-\frac{\pi R}{2}\right)\right)\mathcal{Q}^{a}(A), \tag{4.8}$$

$$(D_M J^M)^{i}(x,y) = \mp\frac{1}{2}\left(\delta(y)-\delta\left(y-\frac{\pi R}{2}\right)\right)\mathcal{Q}^{i}(A)\,, \tag{4.9}$$

$$(D_M J^M)^{B}(x,y) = \pm\frac{1}{2}\left(\delta(y)-\delta\left(y-\frac{\pi R}{2}\right)\right)(\mathcal{Q}^{B}_{-}(A)-\mathcal{Q}^{B}_{+}(A))\,. \tag{4.10}$$

In this case, we have no integrated anomalies either but the structure of localized anomalies are different from the case with two brane fields at different branes.

The existence of the localized gauge anomalies in either type of GUT orbifolds could make the theory with the unbroken gauge group anomalous. However, these localized gauge anomalies can be exactly cancelled with the introduction of a Chern-Simons(CS) 5-form $Q_5[A=A^qT^q]$ with a jumping coefficient in the action [5]

$$\mathcal{L}_{CS} = -\frac{1}{96\pi^2}\epsilon(y)Q_5[A] \tag{4.11}$$

where $\epsilon(y)$ is the sign function with periodicity πR and

$$Q_5[A] = \mathrm{Tr}\left(AdAdA+\frac{3}{2}A^3dA+\frac{3}{5}A^5\right). \tag{4.12}$$

The parity-odd function $\epsilon(y)$ in front of Q_5 is necessary for the parity invariance because Q_5 is a parity-odd quantity according to our parity assignments for bulk gauge fields, eqs. (2.19)-(2.22) for type-I and eqs. (2.27)-(2.30) for type-II. Under the gauge transformation $\delta A = d\omega + [A,\omega] \equiv D\omega$,

$$\delta Q_5 = Q_4^1[\delta A, A] = \mathrm{str}\left(D\omega\, d(AdA + \frac{1}{2}A^3)\right) \tag{4.13}$$

where str means the symmetrized trace and the restricted gauge transformation in eqs. (2.31) and (2.32) is understood. Then, due to the sign function in front of Q_5, the variation of the Chern-Simons action gives rise to the 4D consistent anomalies on the boundaries for type-I and II, respectively,

$$\begin{aligned}\delta\mathcal{L}_{CS} &= \frac{1}{48\pi^2}\left(\delta(y) - \delta\left(y - \frac{\pi R}{2}\right)\right)\epsilon^{\mu\nu\rho\sigma}\sum_{q=a,i,B}\omega^q \mathrm{str}\left(T^q\partial_\mu\left(A_\nu\partial_\rho A_\sigma + \frac{1}{2}A_\nu A_\rho A_\sigma\right)\right) + \\ &\quad + \frac{1}{48\pi^2}\delta(y)\,\epsilon^{\mu\nu\rho\sigma}\omega^{\hat{a}}\mathrm{str}\left(T^{\hat{a}}\partial_\mu\left(A_\nu\partial_\rho A_\sigma + \frac{1}{2}A_\nu A_\rho A_\sigma\right)\right), \end{aligned}\tag{4.14}$$

$$\begin{aligned}\delta\mathcal{L}_{CS} &= \frac{1}{48\pi^2}\left(\delta(y) - \delta\left(y - \frac{\pi R}{2}\right)\right)\epsilon^{\mu\nu\rho\sigma} \times \\ &\quad \times \sum_{q=a,i,B}\omega^q \mathrm{str}\left(T^q\partial_\mu\left(A_\nu\partial_\rho A_\sigma + \frac{1}{2}A_\nu A_\rho A_\sigma\right)\right). \end{aligned}\tag{4.15}$$

The consistent anomalies for type-I we obtained here can be changed to the covariant anomalies [19] by regarding the covariant non-abelian gauge current J^q_μ as being redefined from a non-covariant gauge current $\tilde{J}^q_\mu$ as

$$J^q_\mu(x,y) = \tilde{J}^q_\mu(x,y) + U^q_\mu(x,y) \tag{4.16}$$

where

$$\begin{aligned} U^{q=(a,i,B)}_\mu &= -\frac{1}{96\pi^2}\left(\delta(y) - \delta\left(y - \frac{\pi R}{2}\right)\right)\epsilon^{\mu\nu\rho\sigma}\mathrm{str}(T^q(A_\nu F_{\rho\sigma} + F_{\rho\sigma}A_\nu - A_\nu A_\rho A_\sigma)) \\ U^{q=\hat{a}}_\mu &= -\frac{1}{96\pi^2}\delta(y)\,\epsilon^{\mu\nu\rho\sigma}\mathrm{str}(T^q(A_\nu F_{\rho\sigma} + F_{\rho\sigma}A_\nu - A_\nu A_\rho A_\sigma)). \end{aligned}\tag{4.17}$$

For type-II, only the former one in the above is needed. Consequently, the CS term contributes to the anomaly for the 5D covariant gauge current in type-I as

$$\begin{aligned}(D_M J^M)^{q_1=(a,i,B)} &= -\frac{1}{64\pi^2}\left(\delta(y) - \delta\left(y - \frac{\pi R}{2}\right)\right)\sum_{q_2,q_3=(b,j,B)}\mathrm{str}(T^{q_1}T^{q_2}T^{q_3})F^{q_2}_{\mu\nu}F^{q_3\mu\nu} - \\ &\quad - \frac{1}{64\pi^2}\delta(y)\sum_{q_2,q_3=\hat{a}}\mathrm{str}(T^{q_1}T^{q_2}T^{q_3})F^{q_2}_{\mu\nu}F^{q_3\mu\nu}, \end{aligned}\tag{4.18}$$

$$(D_M J^M)^{q_1=\hat{a}} = -\frac{1}{64\pi^2}\delta(y)\sum_{q_2 q_3=\hat{b}(a,i,B)}\mathrm{str}(T^{q_1}T^{q_2}T^{q_3})F^{q_2}_{\mu\nu}F^{q_3\mu\nu}, \tag{4.19}$$

and in type-II as

$$(D_M J^M)^{q_1=(a,i,B)} = -\frac{1}{64\pi^2}\left(\delta(y) - \delta\left(y - \frac{\pi R}{2}\right)\right) \sum_{q_2,q_3=(b,j,B)} \mathrm{str}(T^{q_1}T^{q_2}T^{q_3})F^{q_2}_{\mu\nu}F^{q_3\mu\nu} \tag{4.20}$$

$$(D_M J^M)^{q_1=\hat{a}} = 0 \tag{4.21}$$

where $q_{1,2,3}$ run the bulk group indices. It turns out that the CS contributions to the anomalies exactly cancel the localized gauge anomalies on the boundaries, eqs. (4.1)–(4.4) in type-I. On the other hand, in type-II, the CS term with a correct overall sign also exactly cancels the localized gauge anomalies, eqs. (4.5) and (4.6), only in the case with two brane fields at different branes but not at the same brane. In the case with two brane fields at the same brane, we find that the bulk CS term is not sufficient for cancelling all localized anomalies from the bulk and brane matter fields.

5. Fayet-Iliopoulos terms

In type-I, the only place where the U(1)-graviton-graviton anomalies could appear is the fixed point $y = \pi R/2$ with the local gauge group including a U(1) gauge factor. On the other hand, in type-II, the mixed gravitational anomalies could appear on both fixed points since their unbroken gauge group has a U(1) gauge factor. As argued in the literature [5], however, there is no gravitational counterpart $A \wedge R \wedge R$ of the 5D Chern-Simons term since the non-abelian gauge fields propagate in the bulk.

It has been shown that the gravitational anomalies at $y = \pi R/2$ in type-I indeed cancel between the bulk and brane contributions without the need of a bulk Chern-Simons term [5]. On the other hand, in type-II, it is only for two brane fields at different branes that there is no localized gravitational anomalies. Then, since both gravitational anomalies and FI terms are proportional to the common factor Tr (q), where q is the U(1) charge operator, it seems that the absence of the gravitational anomalies should guarantee the absence of the FI terms which could also exist at $y = \pi R/2$ in type-I and at both fixed points in type-II. This is the requirement for the stability of the 4D supersymmetric theory.

In the orbifold models with an unbroken U(1), however, it has been shown that the localized FI terms can be induced from a bulk field without breaking the 4D supersymmetry [7, 9, 11, 17]. In this section, we present the explicit computation of the Fayet-Iliopoulos(FI) terms [14, 11] for our set of bulk and brane fields in our model.

The relevant part of the action for bulk(h, h^c) and brane(h_b) scalar fields for performing the FI term calculation in type-I is given by

$$S = \int d^4x \int_0^{2\pi R} dy \Big[|\partial_M h|^2 + |\partial_M h^{c\dagger}|^2 + gD^B(h^\dagger T^B h - h^c T^B h^{c\dagger}) + \\ + \delta\left(y - \frac{\pi R}{2}\right)\left(|\partial_\mu h_b|^2 + gD^B h_b^\dagger q_b h_b\right)\Big] \tag{5.1}$$

where D^B imply the auxiliary field for the unbroken U(1). Denoting the bulk scalar fields as $h = (h_{++}, h_{+-})^T$ and $h^c = (h_{--}, h_{-+})$, let us expand those in terms of bulk eigenmodes as

$$h_{\pm\pm}(x,y) = \sum_n h_{(\pm\pm)n}(x)\xi_n^{(\pm\pm)}(y)\,, \tag{5.2}$$

$$h_{\pm\mp}(x,y) = \sum_n h_{(\pm\mp)n}(x)\xi_n^{(\pm\mp)}(y)\,. \tag{5.3}$$

As in the anomaly computation, inserting the above mode expansions in the 5D action gives

$$\begin{aligned} S = \int d^4x \Big[&\sum_{\alpha,\beta=\pm} \sum_{m,n} -h^\dagger_{(\alpha\beta)m}(x)\Big((\Box_4 + M_n)\delta_{mn} - g q_{\alpha\beta} D^{B(\alpha\beta)}_{mn}(x)\Big) h_{(\alpha\beta)n}(x) - \\ &- h_b^\dagger(x)\left(\Box_4 - g q_{--} D^B\left(x, y = \frac{\pi R}{2}\right)\right) h_b(x)\Big] \end{aligned} \tag{5.4}$$

where

$$D^{B(\alpha\beta)}_{mn}(x) = \int_0^{2\pi R} dy\, \xi_m^{(\alpha\beta)}(y)\xi_n^{(\alpha\beta)}(y) D^B(x,y) \tag{5.5}$$

and $\mathrm{Tr}\,(T^B) = Kq_{++} + Nq_{+-} = 0$, $q_{-+} = -q_{+-}$, $q_{--} = -q_{++}$, and the introduction of a brane $\bar{K}$-plet with $q_b = q_{--}$ is understood.

From the one-loop tadpole diagram for the KK modes of auxiliary field D^B, we can get the bulk and brane field contributions to the FI term with the cutoff Λ regularization as follows

$$F(x) = F_{bulk}(x) + F_{brane}(x) \tag{5.6}$$

where

$$F_{bulk}(x) = \sum_n \sum_{\alpha\beta} q_{\alpha\beta} T_n D^{B(\alpha\beta)}_{nn}(x) \tag{5.7}$$

with

$$T_n = ig \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - M_n^2} = \frac{g}{16\pi^2}\left(\Lambda^2 - M_n^2 \ln \frac{\Lambda^2 + M_n^2}{M_n^2}\right), \tag{5.8}$$

and

$$F_{brane}(x) = igKq_{--} D^B\left(x, y = \frac{\pi R}{2}\right) \int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2} = \frac{gKq_{--}}{16\pi^2}\Lambda^2 D^B\left(x, y = \frac{\pi R}{2}\right). \tag{5.9}$$

Then, when we write the FI term in terms of the 5D field $D^B(x,y)$ as

$$F(x) = \int_0^{2\pi R} dy\, f(y) D^B(x,y)\,, \tag{5.10}$$

we make an inverse Fourier-transformation for the auxiliary field to obtain the bulk profile for the FI term as

$$f(y) = f_{even}(y) + f_{odd}(y) + f_{brane}(y) \tag{5.11}$$

where

$$f_{even}(y) = gKq_{++}\sum_n T_n[|\xi_n^{(++)}|^2 - |\xi_n^{(--)}|^2] \tag{5.12}$$
$$= \frac{gKq_{++}}{16\pi^2}\left[\frac{1}{2}\Lambda^2\left(\delta(y)+\delta\left(y-\frac{\pi R}{2}\right)\right)+\frac{1}{4}\ln\frac{\Lambda}{\mu}\left(\delta''(y)+\delta''\left(y-\frac{\pi R}{2}\right)\right)\right],$$
$$f_{odd}(y) = gNq_{+-}\sum_n T_n[|\xi_n^{(+-)}|^2 - |\xi_n^{(-+)}|^2] \tag{5.13}$$
$$= \frac{gNq_{+-}}{16\pi^2}\left[\frac{1}{2}\Lambda^2\left(\delta(y)-\delta\left(y-\frac{\pi R}{2}\right)\right)+\frac{1}{4}\ln\frac{\Lambda}{\mu}\left(\delta''(y)-\delta''\left(y-\frac{\pi R}{2}\right)\right)\right],$$
$$f_{brane}(y) = \frac{gKq_{--}}{16\pi^2}\Lambda^2\delta\left(y-\frac{\pi R}{2}\right). \tag{5.14}$$

Here, prime denotes the derivative with respect to the extra dimension coordinate. Consequently, the resultant FI term in type-I is given by

$$f(y) = \frac{g\mathrm{Tr}\,(T^B)}{32\pi^2}\left[\Lambda^2\left(\delta(y)+\delta\left(y-\frac{\pi R}{2}\right)\right)+\frac{1}{2}\ln\frac{\Lambda}{\mu}\delta''(y)\right]+\frac{gKq_{++}}{32\pi^2}\ln\frac{\Lambda}{\mu}\delta''\left(y-\frac{\pi R}{2}\right)$$
$$= \frac{gKq_{++}}{32\pi^2}\ln\frac{\Lambda}{\mu}\delta''\left(y-\frac{\pi R}{2}\right) \tag{5.15}$$

where we used $\mathrm{Tr}\,(T^B)=0$ in the last line. We note that there is no FI term at $y=0$ with the full bulk gauge group, which is as expected because there is no U(1) factor at this fixed point. Moreover, we find that there is no conventional FI term with quadratic divergence even at $y=\pi R/2$ with a U(1) factor, which is consistent with the absence of mixed gravitational anomalies as argued in [5]. However, there exists a non-vanishing FI term with logarithmic divergence at $y=\pi R/2$.

Likewise, following the same procedure, we find that the localized FI terms in type-II depend on the location of brane fields: with both $\bar{K}$-plet and N-plet at the same brane $y=0(y=\pi R/2)$ as

$$f(y) = \pm\frac{gKq_{++}}{16\pi^2}\Lambda^2\left(\delta(y)-\delta\left(y-\frac{\pi R}{2}\right)\right)+\frac{gKq_{++}}{32\pi^2}\ln\frac{\Lambda}{\mu}\left(\delta''(y)+\delta''\left(y-\frac{\pi R}{2}\right)\right) \tag{5.16}$$

and with a $\bar{K}$-plet at $y=0(y=\pi R/2)$ and a N-plet at $y=\pi R/2(y=0)$ as

$$f(y) = \frac{gKq_{++}}{32\pi^2}\ln\frac{\Lambda}{\mu}\left(\delta''(y)+\delta''\left(y-\frac{\pi R}{2}\right)\right). \tag{5.17}$$

In the case with two brane fields at the same brane, there appear quadratic FI terms as well as log FI terms. However, in this case, even if the integrated FI terms are zero, the localized FI terms with quadratic divergence would not give rise to a consistent theory at low energies since there is no gravitational CS term to cancel the mixed gravitational anomalies, $\pm Kq_{++}$ at $y=0$ and $\mp Kq_{++}$ at $y=\pi R$. On the other hand, in the case with two brane fields at different branes, there are only log divergent FI terms at both fixed points which maintain the 4D supersymmetry and the mass spectrum of a bulk multiplet [11].

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6. Conclusion

We considered the breaking of the 5D non-abelian gauge symmetry on $S^1/(Z_2 \times Z_2')$ orbifold. Then, we presented the localized gauge anomalies coming from a bulk fundamental field through the explicit KK mode decomposition of the 5D fields. In the orbifold with gauge symmetry breaking, there are fixed points with their own local gauge symmetries. Thus, there is the possibility of embedding some incomplete multiplets at the fixed point with unbroken gauge group, which can be sometimes phenomenologically preferred. The incomplete brane multiplet we considered can be realized from a bulk muliplet in the field theoretic limit. Therefore, we have shown that the 4D anomaly combination of a brane field and a bulk zero mode does not have the localized gauge anomalies up to the addition of a Chern-Simon 5-form with some jumping coefficient, which could be regarded as the effects of the bulk heavy modes as in the abelian gauge theory on S^1/Z_2. However, for the brane fields assigned at the same fixed points in models of type-II, we found that it is not possible to cancel the localized anomalies only with the CS term even if there is no 4D anomaly.. Then, we also found a nonzero log FI term at the fixed point with H in both types of orbifold boundary conditions we considered.

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